

Safe and Efficient Navigation in Dynamic Environments

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Research Focus: Navigation in Dynamic Environments

Dynamic Environments: Presence of other dynamic agents

Scripted dynamic agents ¹

Unscripted dynamic agents ²

Objective

Given a dynamic environment, a start location and a goal, find a

- ▶ safe,
- ▶ dynamically feasible,
- ▶ bounded cost suboptimal path for the robot

Challenges

To navigate in dynamic environments, the robot needs to

1. **quickly plan** a path accounting for dynamic agents
2. have an **accurate model** of the dynamics of environment

In this thesis, we will tackle both challenges

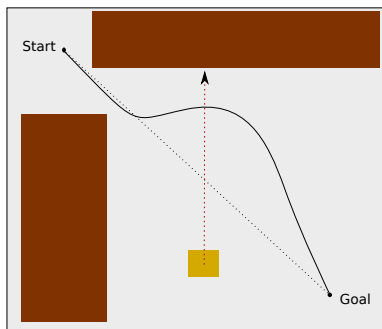
Background

Planning in Dynamic Environments with Adaptive Dimensionality

Modeling Cooperative Navigation in Crowds

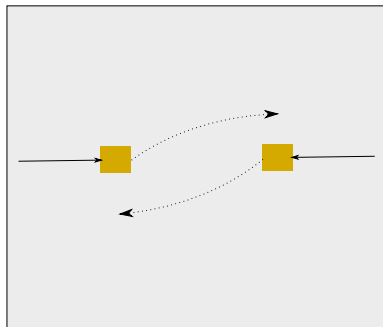
Conclusion and Future Work

Path Planning



Find a path for the robot that avoids collisions with static and dynamic obstacles, and is dynamically feasible to execute

Trajectory Prediction



Given the past trajectories of dynamic agents in the environment,
predict their future trajectories until a fixed time horizon

Background

Planning in Dynamic Environments with Adaptive Dimensionality

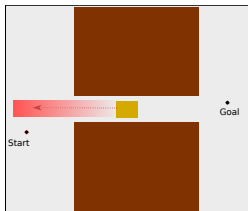
Modeling Cooperative Navigation in Crowds

Conclusion and Future Work

Problem Statement

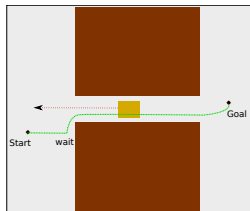
Assuming a perfect model of world dynamics **exists**, given a start and goal configuration, find a path that is **safe** and **feasible** to execute, with **bounded cost suboptimality**

Past Work : Planning in Dynamic Environments



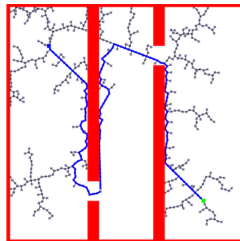
Planning without
time dimension³

Incomplete



Safe Interval Path
Planning⁴

Wait in-place

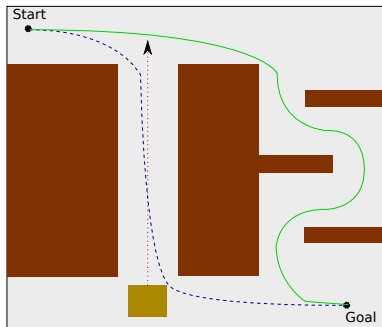


RRT⁵

No optimality
guarantees

Motivation

Low-D heuristics tend to be wrong in dynamic environments



The solution path (green) is against heuristic

Hence, heuristic-based planners are **extremely slow**

Adaptive Dimensionality Graph

Core Idea : Consider time dimension **only** in regions where there is a potential dynamic obstacle collision. Plan in low-dimension elsewhere.

Example AD graph

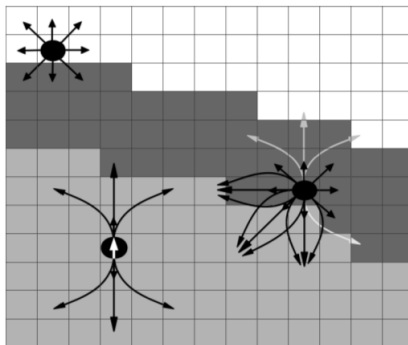


Figure : Low-D: (x, y) , High-D: (x, y, θ) ⁶

Extending to Planning in Dynamic Environments

State-spaces

- ▶ S^{ld} : Only spatial variables e.g. (x, y)
- ▶ S^{hd} : Spatio-temporal variables e.g. (x, y, t)
- ▶ Bound the time dimension in S^{hd} by an upper bound T

Transition sets

- ▶ T^{ld} : Transitions between states in S^{ld}
- ▶ T^{hd} : Transitions between states in S^{hd}
- ▶ Collision with dynamic obstacle **only checked** in T^{hd}

Extending to Planning in Dynamic Environments

Projections

- ▶ Projection function $\lambda : S^{hd} \rightarrow S^{ld}$
- ▶ Inverse projection function λ^{-1} is given by:

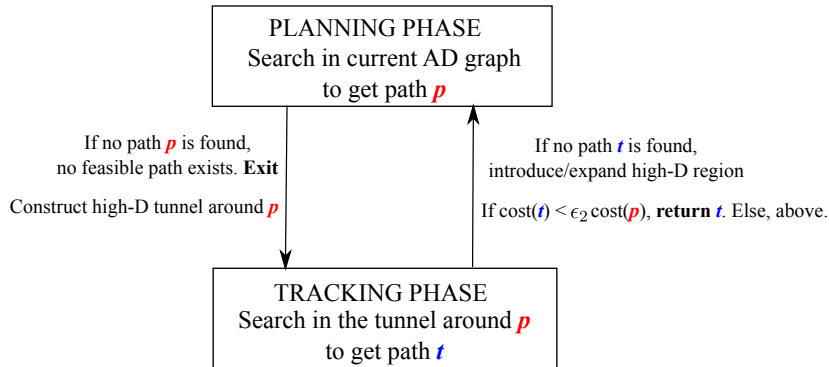
$$\lambda^{-1}(X^{ld}) = \{X^{hd} \mid \lambda(X^{hd}) = X^{ld}, t_d(X^{ld}) \leq t(X^{hd}) \leq T\}$$

- ▶ We obtain t_d using a time-optimal Dijkstra search in G^{ld} to find the least time taken to reach any state from start

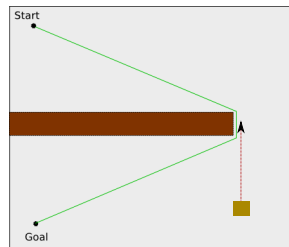
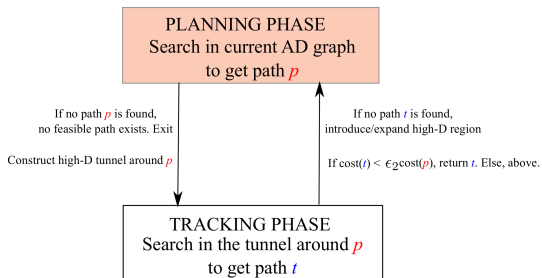
Adaptive transitions

- ▶ $(X^{hd}, Y^{ld}) \in T^{ad}$, if $Y^{ld} = \lambda(X^{hd})$
- ▶ $(X^{ld}, Y^{hd}) \in T^{ad}$, if $(X^{hd}, Y^{hd}) \in T^{hd}$ and $X^{hd} \in \lambda^{-1}(X^{ld})$

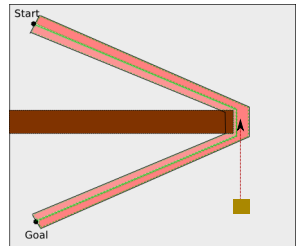
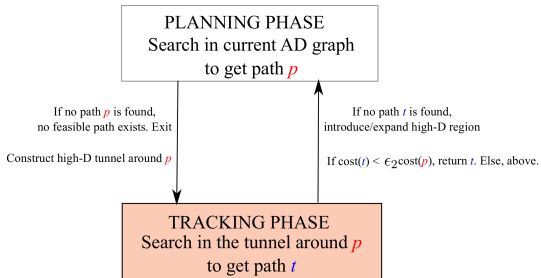
Main Loop



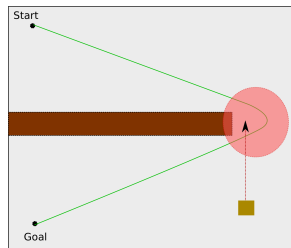
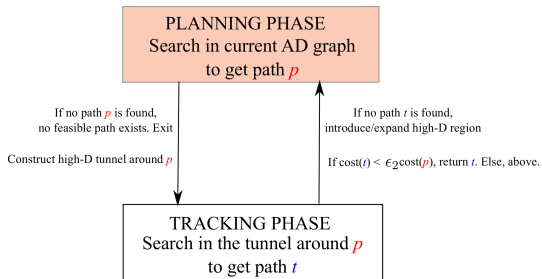
Main Loop : Start with $G^{ad} = G^{ld}$



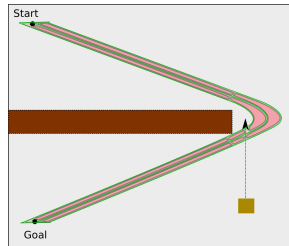
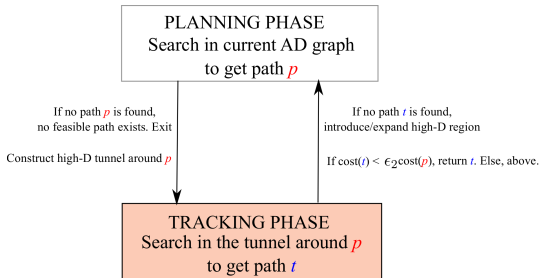
Main Loop : Construct High-D tunnel around path



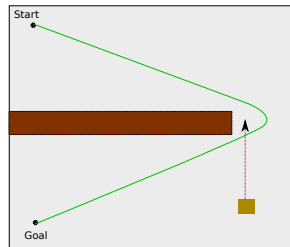
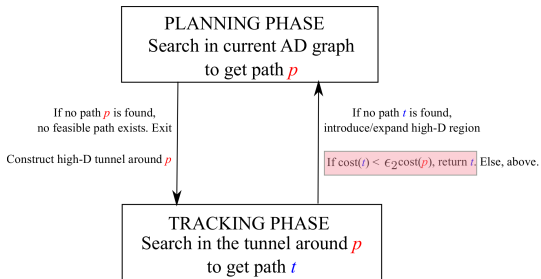
Main Loop : Add High-D region and search G^{ad}



Main Loop : No collisions in High-D tunnel



Main Loop : Success



Theoretical Properties

Theorem 1 : Completeness

If no path $\pi_{G^{hd}}$ is found in planning phase, no collision-free, feasible path exists that can reach goal from start within time T in G^{hd}

Theorem 2 : Bounded cost suboptimality

If a weighted A* is used for both planning and tracking phase, then the cost of resulting path (if found) has cost no more than $\epsilon_{plan} \cdot \epsilon_{track} \cdot c(\pi_{G^{hd}}^*(X_S, X_G))$

3D Nonholonomic Robot

- ▶ Simulated 3D (x, y, θ) robot in randomly generated dynamic environments
- ▶ Hence, S^{hd} is a 4D- (x, y, θ, t) space. We chose S^{ld} to be the 2D- (x, y) space
- ▶ Cost of a path is the time taken to execute the path
- ▶ We compare our performance against 4D- (x, y, θ, t) HCA* with 2D- (x, y) Dijkstra heuristic

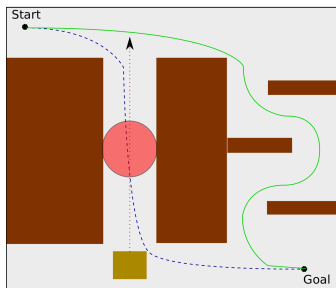
Quantitative Results

Algorithm	Number of Success (Out of 50)	Epsilon ($\epsilon_{plan} \cdot \epsilon_{track}$)	time (secs)	
			mean	std dev
Adaptive	41	1.1	6.7	0.8
HCA*	5	1.1	91.0	71.2
Adaptive	43	1.5	11.7	14.0
HCA*	21	1.5	70.3	86.7
Adaptive	46	2.0	18.5	26.6
HCA*	23	2.0	35.8	69.8

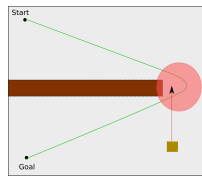
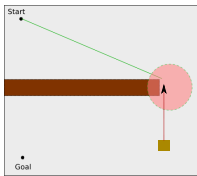
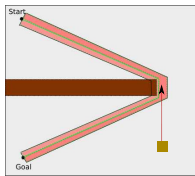
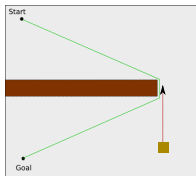
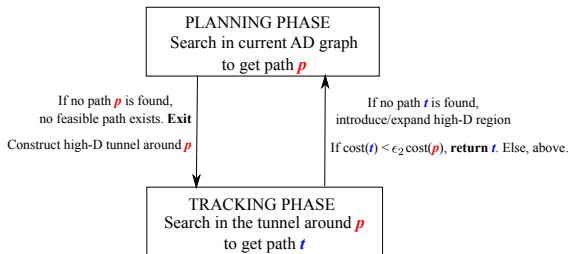
Table : Results on 50 environments with 30 dynamic obstacles.

Discussion

- ▶ Path suggested by Low-D heuristic blocked by dynamic obstacles, our approach **outperforms** baseline significantly
- ▶ Otherwise, we have comparable performance with baseline



Summary



Outline

Background

Planning in Dynamic Environments with Adaptive Dimensionality

Modeling Cooperative Navigation in Crowds

Conclusion and Future Work

Problem Statement

Given sensory information regarding the past trajectories of agents in a dynamic environment, **predict** their future trajectories

In this work, we will focus on human crowds

Cooperation in Crowds

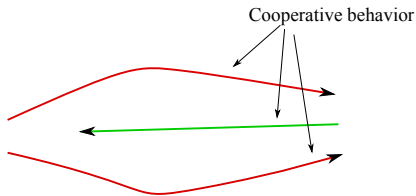
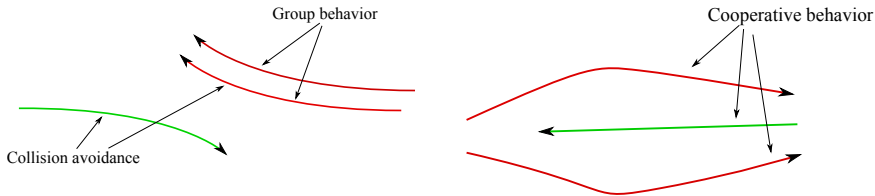
Sparse⁷

Moderate⁸

Dense⁸

Humans navigate through crowds by **adapting** their trajectories to those of other people in the vicinity

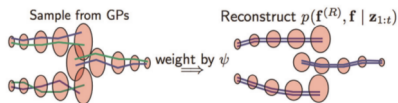
Crowd interactions



To capture such interactions, we need to model the **joint distribution** of trajectories

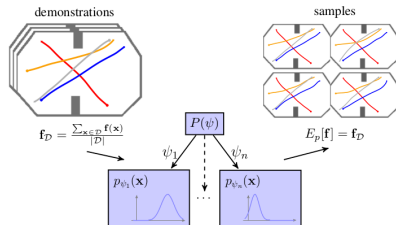
But, a joint distribution model scales with the number of agents in the crowd

Past Work : Navigation in Human Crowds



Interacting Gaussian Processes³

Handcrafted potential term



Inverse Reinforcement Learning⁸

Scales poorly

Notation

- ▶ $\mathbf{f}^{(i)}$: Trajectory of agent i , a sequence of (x, y) locations for discrete time-steps 1 to T
- ▶ $\mathbf{z}_{1:t}^{(i)}$: Observed locations of agent i at time-steps 1 to t
- ▶ $\mathbf{z}_{1:t}$: Observed locations of all agents
- ▶ \mathbf{f} : Trajectories of all agents
- ▶ $(\mathbf{v}_x)_t^{(i)}, (\mathbf{v}_y)_t^{(i)}$: x- and y-velocity of agent i at time t

Note: We assume there are a **fixed set** of goals G in the environment

Joint Prediction Model

Independent model :

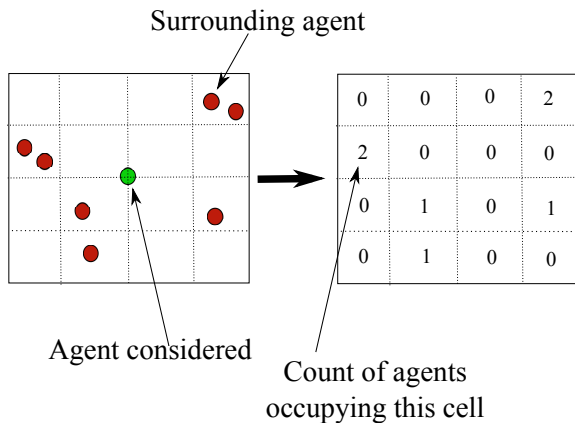
$$P(\mathbf{f}|\mathbf{z}_{1:t}) = \prod_{i=1}^N P(\mathbf{f}^{(i)}|\mathbf{z}_{1:t}^{(i)})$$

Interacting Gaussian processes (Trautman and Krause 2015) :

$$P(\mathbf{f}|\mathbf{z}_{1:t}) = \frac{1}{Z} \underbrace{\psi(\mathbf{f}^{(1)}, \dots, \mathbf{f}^{(N)})}_{\text{Handcrafted potential}} \prod_{i=1}^N P(\mathbf{f}^{(i)}|\mathbf{z}_{1:t}^{(i)})$$

Instead of handcrafted potential, we couple predictions \mathbf{f} using
Occupancy Grids

Occupancy Grids



Scaling up using occupancy grids

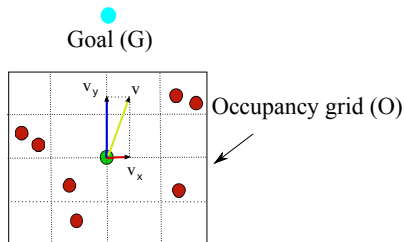
Given observed part of trajectories $\mathbf{z}_{1:t}$,

$$\begin{aligned} P(\mathbf{f}|\mathbf{z}_{1:t}) &= \sum_{\mathbf{g}} P(\mathbf{f}|\mathbf{g}, \mathbf{z}_{1:t})P(\mathbf{g}|\mathbf{z}_{1:t}) \\ &= \sum_{\mathbf{g}} \prod_{i=1}^N \underbrace{P(\mathbf{f}^{(i)}|\mathbf{O}_{1:t}^{(i)}, \mathbf{g}^{(i)}, \mathbf{z}_{1:t}^{(i)})}_{\text{prediction}} \underbrace{P(\mathbf{g}^{(i)}|\mathbf{z}_{1:t}^{(i)})}_{\text{inferring goal}} \end{aligned}$$

where $\mathbf{O}_t^{(i)}$ is the occupancy grid of agent i at time t and $\mathbf{g}^{(i)}$ is the goal location of agent i .

Training Local Interaction Model

For each goal $\mathbf{g} \in G$, where G is the set of goals in the environment we estimate the distribution $P(\mathbf{v}_x | \mathbf{O}, \mathbf{g})$ and $P(\mathbf{v}_y | \mathbf{O}, \mathbf{g})$ using Gaussian Process regression



Learn $P(v|O, G)$ from training data

Goal Inference

$$P(\mathbf{f}|\mathbf{z}_{1:t}) = \sum_{\mathbf{g}} \prod_{i=1}^N \underbrace{P(\mathbf{f}^{(i)}|\mathbf{O}_{1:t}^{(i)}, \mathbf{g}^{(i)}, \mathbf{z}_{1:t}^{(i)})}_{\text{prediction}} \underbrace{P(\mathbf{g}^{(i)}|\mathbf{z}_{1:t}^{(i)})}_{\text{inferring goal}}$$

At inference time,

$$\begin{aligned} P(\mathbf{g}^{(i)}|\mathbf{z}_{1:t}^{(i)}) &\propto P(\mathbf{z}_{1:t}^{(i)}|\mathbf{g}^{(i)})P(\mathbf{g}^{(i)}) \quad \text{Bayes rule} \\ &\propto \underbrace{P((\mathbf{v}_x)_{1:t}^{(i)}|\mathbf{O}_{1:t}^{(i)}, \mathbf{g}^{(i)})P((\mathbf{v}_y)_{1:t}^{(i)}|\mathbf{O}_{1:t}^{(i)}, \mathbf{g}^{(i)})}_{\text{From trained model}} \end{aligned}$$

Trajectory Prediction

$$P(\mathbf{f}|\mathbf{z}_{1:t}) = \sum_{\mathbf{g}} \prod_{i=1}^N \underbrace{P(\mathbf{f}^{(i)}|\mathbf{O}_{1:t}^{(i)}, \mathbf{g}^{(i)}, \mathbf{z}_{1:t}^{(i)})}_{\text{prediction}} \underbrace{P(\mathbf{g}^{(i)}|\mathbf{z}_{1:t}^{(i)})}_{\text{inferring goal}}$$

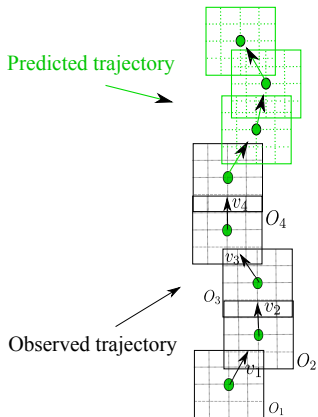
For each time-step $t' > t$, we sample from the predictive GP distribution

$$P((\mathbf{v}_x)_{t'}^{(i)} | \mathbf{O}_{t'}^{(i)}, \mathbf{g}^{(i)}, (\mathbf{v}_x)_{1:t}^{(i)}, \mathbf{O}_{1:t}^{(i)})$$

$$P((\mathbf{v}_y)_{t'}^{(i)} | \mathbf{O}_{t'}^{(i)}, \mathbf{g}^{(i)}, (\mathbf{v}_y)_{1:t}^{(i)}, \mathbf{O}_{1:t}^{(i)})$$

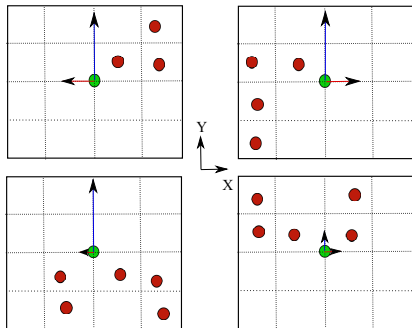
to compute future locations $\mathbf{f}_{t'}$ and occupancy grids $\mathbf{O}_{t'}$ for all agents

Trajectory Prediction



Predict velocities and compute
occupancy grids at each time-step

Results: Learned Behaviors



Collision avoidance



Cooperative behavior

Qualitative Results

IGP

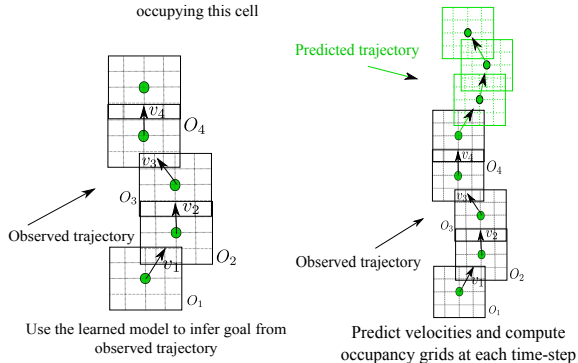
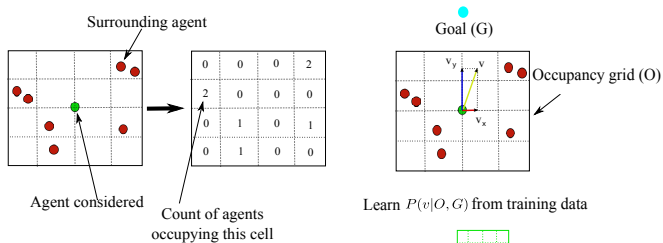
Our approach

Quantitative Results

Table : Prediction errors (in pixels) for IGP and our approach

Metric	Prediction horizon (H)	IGP	Our Approach
Avg. Disp. Error	1	3.42	4.42
	2	5.66	6.14
	5	15.75	12.09
	10	21.59	18.23
	20	41.51	34.63
Final Disp. Error	1	3.42	4.42
	2	7.12	7.78
	5	23.18	19.77
	10	38.75	28.31
	20	67.41	54.2

Summary



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Conclusion

In this thesis, we have successfully explored and answered the following two questions:

1. Given a model of world dynamics, how to obtain safe, feasible and bounded sub-optimal paths for robots in dynamic environments? **Path Planning with Adaptive Dimensionality**
2. How do you model complex world dynamics such as cooperative behavior in dynamic environments, specifically human crowds? **Occupancy grid-based joint modeling**

Path Planning

- ▶ Verify the performance on a mobile robot in a scripted dynamic environment
- ▶ Make planner incremental - reuse search tree from previous iterations
- ▶ Account for uncertainty in the world dynamics model predictions to obtain frequency of replanning

Modeling World Dynamics

- ▶ Account for static obstacles within the model
- ▶ Relax the assumption of fixed goals in the environment
- ▶ Local interaction assumption is not necessarily true. Attend to important surrounding agents (we have recently submitted a work in this direction)

Thank you. Questions?

Publications:

1. Anirudh Vemula, Katharina Muelling, Jean Oh. **Path Planning in Dynamic Environments with Adaptive Dimensionality**. SoCS 2016
2. Anirudh Vemula, Katharina Muelling, Jean Oh. **Modeling Cooperative Navigation in Dense Human Crowds**. ICRA 2017
3. Anirudh Vemula, Katharina Muelling, Jean Oh. **Social Attention: Modeling Attention in Human Crowds**. Submitted to CoRL 2017

¹https://www.youtube.com/watch?v=z_R8feyCu-M

²<https://www.youtube.com/watch?v=dPSxgQgPv28>

³Trautman, Peter, and Andreas Krause. "Unfreezing the robot: Navigation in dense, interacting crowds." *Intelligent Robots and Systems (IROS), 2010 IEEE/RSJ International Conference on*. IEEE, 2010.

⁴Likhachev, Maxim, and Dave Ferguson. "Planning long dynamically feasible maneuvers for autonomous vehicles." *The International Journal of Robotics Research* 28, no. 8 (2009): 933-945.

⁵Phillips, Mike, and Maxim Likhachev. "Sipp: Safe interval path planning for dynamic environments." In *Robotics and Automation (ICRA), 2011 IEEE International Conference on*, pp. 5628-5635. IEEE, 2011.

⁶Bekris, Kostas E., and Lydia E. Kavraki. "Greedy but safe replanning under kinodynamic constraints." In *Robotics and Automation, 2007 IEEE International Conference on*, pp. 704-710. IEEE, 2007.

⁷Gochev, Kalin, Benjamin Cohen, Jonathan Butzke, Alla Safonova, and Maxim Likhachev. "Path planning with adaptive dimensionality." In *Fourth annual symposium on combinatorial search*. 2011.

⁸Lerner, Alon, Yiorgos Chrysanthou, and Dani Lischinski. "Crowds by example." In *Computer Graphics Forum*, vol. 26, no. 3, pp. 655-664. Blackwell Publishing Ltd, 2007.

⁹Kretzschmar, Henrik, Markus Spies, Christoph Sprunk, and Wolfram Burgard. "Socially compliant mobile robot navigation via inverse reinforcement learning." *The International Journal of Robotics Research* 35, no. 11 (2016): 1289-1307.